Problem Set #4
Due: Tuesday, Dec. 15, 2016

1. *Converse for Wyner-Ziv:* The rate distortion function for lossy source coding with non-causal side information, where the source X and side information Y (at the decoder) are an i.i.d. multi-source distributed according to $P_{X,Y}$, is given as follows:

$$R(D) = \min_{P_{U|X}, P_{\hat{X}|Y,U} \,:\, \mathbb{E}d(X,\hat{X}) \leq D} \left(I(U;X) - I(U;Y) \right). \tag{1}$$

   Prove the converse for this claim, which is that the capacity $R(D)$ is greater than or equal to the right side of (1). As an extra challenge (optional), can you also claim the restriction that $\hat{X}$ is a function of Y and U. This will likely fall immediately out of your converse proof if you look carefully (I mean it this time.).

2. *Causal side information at the decoder:* Consider the above Wyner-Ziv setting for lossy compression (with side information at the decoder), except that this time the side information is used causally. That is, for each i, $\hat{X}_i$ must be reconstructed from M (the message) and Y^i.

   For this setting, the rate distortion function completely forfeits one of the two efficiencies that the side information provides, as discussed in the lecture. Decide which efficiency if forfeited, state the new rate-distortion function, and prove achievability and converse (the achievability proof may be done informally if you wish).

3. *Key agreement for erasure source:* Let X and Y be correlated in the following way. X is a binary symmetric variable, and Y is equal to X with random erasures (i.e. X passed through a symmetric erasure channel) with erasure probability P_e. Terminal A receives X^n (i.i.d. across the sequence), and Terminal B receives Y^n.

   (a) What is the key capacity if Terminal A can send a message to Terminal B at rate R?

   (b) What is the key capacity if Terminal B can send a message to Terminal A at rate R?

4. *Noise for Differential Privacy:*

   (a) Let Z have a Laplace distribution. Let $X_1 = Z$ and $X_2 = Z + c$, where c is a constant. Can you bound the ratio of densities

$$\left| \log \frac{f_{X_1}(x)}{f_{X_2}(x)} \right| \tag{2}$$

   for all x?

   (b) Repeat the above with Z having a Gaussian distribution.